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COMPLEXITY EXAMINATION OF SURE-P, SURE-LS AND REVE SINGLE RULE REASONING METHODS

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ABSTRACT:
Non conventional Fuzzy systems – whose rule base is spare – do not have rules for all the possible observations, in other words it could exists one (or more) observation(s) that does not lead to an interpretable conclusion applying the classical fuzzy reasoning. In this case the conventional fuzzy inference methods (Zadeh, Mamdani, Larsen, or Takagi-Sugeno cannot be used; one has to apply a Fuzzy Rule Interpolation Technique (FRIT).
Complexity is a feature that in several cases determines the practical applicability of the FRITs. Despite its importance one barely can find a paper in the literature, which covers this topic. This paper tries to fill this gap by analyzing the complexity of three single rule reasoning methods (SURE-p, SURE-LS and REVE) that were developed for the second step of the generalized methodology of fuzzy rule interpolation.

KEYWORDS:
fuzzy rule interpolation, complexity, single rule reasoning methods

1. INTRODUCTION

Fuzzy systems applying a sparse rule base often are considered as non-conventional ones. The term sparse means that the antecedent sets of the rule do not ensure a full coverage of the input space and therefore for some observations there are no rules that could be fired in order to calculate the output (conclusion) of the system. This is why the classical fuzzy reasoning methods (e.g. Zadeh [13], Mamdani [9], Takagi-Sugeno [12]) based on the fuzzy intersection between the observation sets and the rule antecedent sets cannot always ensure an interpretable conclusion.

The problem is solved by the fuzzy inference techniques based on rule interpolation. However, their practical applicability is strongly related to their computational complexity in several cases. Although numerous papers evaluate and survey different fuzzy rule interpolation techniques (FRITs) they mainly concentrate on other properties. For example Mizik, Baranyi, Korondi and Kóczy [10] as well as Mizik, Szabó and Korondi analyze the connection between the fuzziness of the observation and the fuzziness of the conclusion in case of four FRI methods [11]. Johanyák and Kovács [2] survey and evaluate a wider range of FRI methods based on a self defined application oriented evaluation criteria set. Johanyák and Kovács also published and applied in [5] a specialized condition set aiming the characterization of single rule reasoning methods.

This paper tries to fill this gap by analyzing the computational complexity of three single rule reasoning methods (SURE-p [3], SURE-LS [4] and REVE [6]) that were developed for the second step of the generalized methodology of fuzzy rule interpolation (GM) [1]. The rest of this paper is organized as follows. Section 2 recalls the basic concepts of the GM. Section 3 introduces the main idea of the examination. The individual methods are analyzed in Sections 4,5 and 6.

2. GENERALIZED METHODOLOGY OF FUZZY RULE INTERPOLATION

Baranyi, Kóczy and Gedeon defined in [1] a two-step methodology called generalized methodology of fuzzy rule interpolation (GM) for the determination of the conclusion in sparse fuzzy rule bases. The main concept is the following:

Firstly a new rule is interpolated (or extrapolated) in that point of the antecedent space, which is defined by the reference points of the observation sets in each antecedent partition. This step consists of three phases:

- determination of the antecedent sets of the new rule (set interpolation),
- determination of reference points of the consequent sets of the new rule,
- determination of the shape of the consequent sets of the new rule (set interpolation)

Secondly the conclusion is determined by firing the interpolated rule using a Single Rule Reasoning (SRR) method. SRR methods calculate the conclusion by modifying the consequent sets of the new rule. The measure of the modification is related to the dissimilarities/differences between the rule antecedent sets and the observation sets.

3. EXAMINATION

There are more possibilities for examining the methods' goodness. One can prognosticate the steps of running with help of deductive logic or probability calculus or algorithmic description. Complexity is the intrinsic minimum amount of resources, for instance, memory, time, messages, etc., needed to solve a problem or execute an algorithm. There are two main types of complexity measurement, examination: time complexity and space complexity. The complexity depends on "size" of the input. One has introduced several types of asymptotic notation which are used to compare the performance and efficiency of algorithms.

Definition Let g(n) be a function. The set O(g(n)) is defined as O(g(n)) ={ f(n) there exist constant c > 0, n0 >0, such that $0 \le f(n) \le cg(n)$ for all n > n0 }. In other words, f (n) = O(g(n)) if and only if there exist positive constants c, and n0 , such that for all n > n0, the inequality $f(n) \le cg(n)$ is satisfied. g(n) is an asymptotic upper bound for f (n).

4. SURE-p method

The Single rUle REasoning based on polar cuts (SURE-p) was introduced by Johanyák and Kovács in [3] as a complement method of the set interpolation technique FEAT-p [3]. The main idea of this method is the following: in case of each polar level the polar distance of the rule consequent should be modified by the average difference between the polar distances of the rule antecedent and observation sets. One can distinguish four steps during the calculations:

I. Normalize all partitions to the unit interval in all dimensions.

II. Calculate the differences of polar distances ($\rho(A_i^*,\theta)$ and $\rho(A_i^i,\theta)$ on figure 1) for all polar levels on the antecedent side (depends on the predefined polar resolution)

$$r_i(\theta) = \rho(A_i^i,\theta) - \rho(A_i^*,\theta), \tag{1}$$

where *i* is the ordinal number of the polar cut.

III. Calculate the average difference

$$\bar{r}(\theta) = \frac{\sum_{i=1}^{N} r_i(\theta)}{N}, \tag{2}$$

where ND is the number of dimensions.

IV. Apply the calculated modification value on the consequent side

$$\rho(B^i,\theta) - \rho(B^*,\theta) = \bar{r}(\theta). \tag{3}$$

The result of the equation (3) could be invalid, so one has to apply a correction formula of form

$$\rho(B^*,\theta) = \begin{cases} Min\left[\rho(B^i,\theta) - \bar{r}(\theta), \frac{1}{\sin(\theta)}\right] & if\ \theta \in (0,\pi) \\ \rho(B^i,\theta) - \bar{r}(\theta) & if\ \theta = 0\ or\ \theta = \pi. \end{cases} \tag{4}$$

The complexity of this correction is NPC (the complete correction method details can be found in [7]), where NPC is the number of polar cuts. The total complexity of method is the sum

$$NP + ND*NPC + ND + NPC + NPC = NP + ND*NPC + ND + 2NPC, \tag{5}$$

where NP is the number of partitions and ND is the number of dimensions. Thus the complexity class is polynomial quadratic $O(n^2)$.

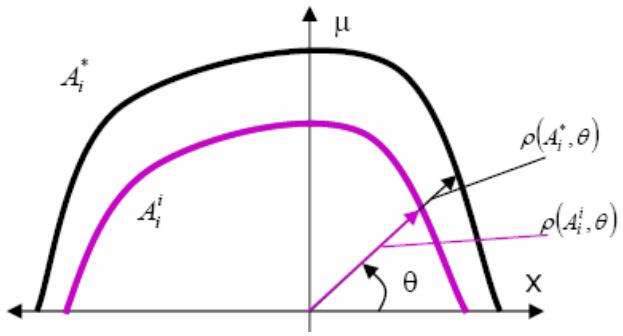

FIGURE 1. THE OBSERVATION (A_i^*) AND ANTECEDENT SET (A_i^i) IN THE *i*th INPUT DIMENSION

5. SURE-LS method

The SURE-LS method proposed by Johanyák and Kovács in [4] was originally developed as a tool for the second step of the FRI method LESFRI [4]. Thus it is complement of the set interpolation technique FEAT-LS [4]. The main idea of this

method is the following: for each a-level the distances between the endpoints of the a-cuts of the conclusion and the rule consequent should be equal to the calculated average horizontal distances between the corresponding endpoints of the a-cuts of the observation and rule antecedent sets. One can distinguish five steps during the calculations:

I. Normalize all partitions to the unit interval in all dimensions.

II. Determine the required number of a levels

$$\Lambda = \left(\bigcup_{i=1}^{ND} \Lambda_i^a \right) \bigcup \Lambda^c \,, \tag{6}$$

where ND is the number of dimensions. In the worst case this number is the sum of all set's break points.

III. In each antecedent dimension for every a-cut calculate the difference

$$d_{i\alpha}^{aL} = \inf \left\{ \left[A_i^i \right]_\alpha \right\} - \inf \left\{ \left[A_i^* \right]_\alpha \right\}. \tag{7}$$

IV. Calculate the average for each a-cut:

$$d_\alpha^{aL} = \frac{\sum_{i=1}^{ND} d_{i\alpha}^{aL}}{ND} \tag{8}$$

V. Apply the difference on the consequent side:

$$d_\alpha^{aL} = \inf \left\{ \left[B^i \right]_\alpha \right\} - \inf \left\{ \left[B^* \right]_\alpha \right\} \tag{9}$$

Supposing that NP is the number of partition, ND is the number of dimensions and NAL is the number of a-levels the total complexity of the method is the following sum:

$$\text{NP} + 1 + \text{ND*NAL} + 1 + \text{NAL}. \tag{10}$$

Thus the complexity class is polynomial quadratic O(n2).

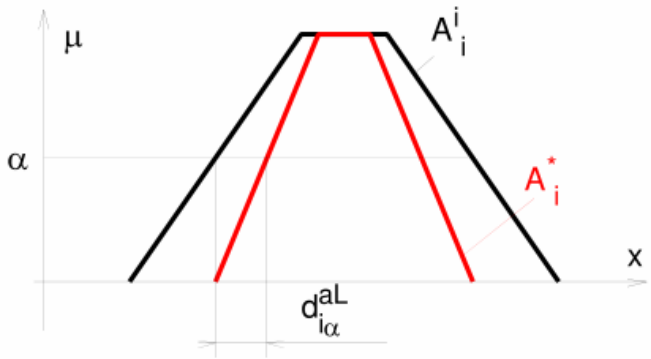

FIGURE 2. THE OBSERVATION (A_i^*) AND ANTECEDENT SET (A_i^i) IN THE *i*th INPUT DIMENSION

6. REVE method

The method REVE proposed by Johanyák and Kovács [6] is based on the concept of Vague Environment (VE) originally introduced by Klawonn [8]. It takes advantage of the partition describing capability of the VE, i.e. in the VE a fuzzy partition is defined by a scaling function and a set of prototype points that correspond to the midpoints of the cores of the fuzzy sets.

REVE is suggested as a complementary of the fuzzy set interpolation method VESI. Therefore in course of the complexity evaluation we suppose that there exists already a pregenerated VE for the antecedent and consequent partitions. This assumption results in lower computational cost.

The main idea of REVE is the conservation of the scaling function ratio in single rule reasoning. The steps of the algorithm are the following:

I. Calculate the scaling functions of the antecedent ($S_{A_i}(x)$) and consequent ($S_{B_i}(x)$) partitions.

II. Calculate the ratio of observation's scaling functions and actual antecedent partition's scaling function for every antecedent dimension

$$r_{A_i}(x) = \frac{S_{A_i^*}(x)}{S_{A_i}(x)} \tag{11}$$

III. Calculate the harmonic mean

$$mr_A(x) = \frac{ND}{\sum_{i=1}^{N} \frac{1}{r_{A_i}(x)}} \; . \tag{12}$$

IV. Determine the scaling function of the conclusion ($S_{B^*}(x)$)

$$S_{B^*}(x) = S_B(x) \cdot mr_A(x) \tag{13}$$

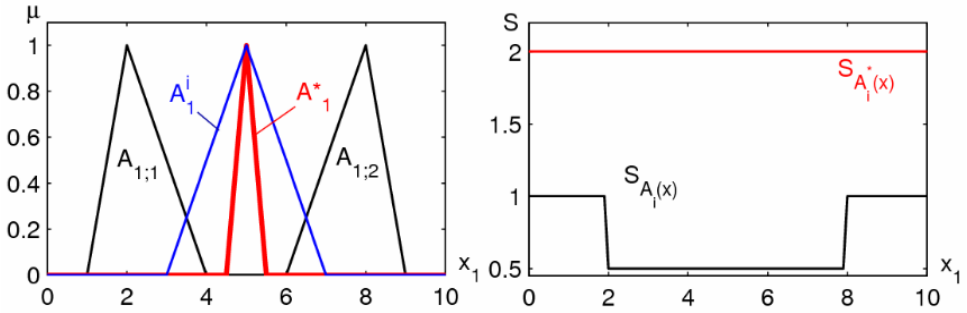

FIGURE 3. THE OBSERVATION (A_i^*) AND ANTECEDENT SET (A_i^i) IN THE *i*th INPUT DIMENSION IN THE LEFT SIDE, AND SCALING FUNCTIONS IN THE RIGHT SIDE

Supposing that NP is the number of partitions, ND is the number of dimensions and NBP is the number of break points the complexity of step I is (2NP+ND) NBP. However, assuming the application of the VESI method before REVE the total complexity of the method becomes the sum of the complexities of parts II to IV

$$ND + ND + NBP, \quad (14)$$

and the complexity class becomes linear O(n).

7. CONCLUSIONS

The examination of computational complexity of FRITs has not been covered sufficiently by previously published papers. This paper concentrates on techniques applicable in the secon step of FRITs following the concepts of GM.

As a result of the examination one can state that the complexity of all methods is polinomic. The REVE method is linear the SURE-p and SURE-LS are quadratic.

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